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MULTI-CHARACTER SENSORY EVALUATION IN PAIRED COMPARISONS

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SUMMARY

A model for multivariate paired comparisons has been evolved for sensory evaluation. A test statistic has been developed for testing the equality of treatment effects on the basis of a number of characters and the null distribution of the test statistic has been worked out. Asymptotically the distribution is observed to be chi-square. The procedure is quite general and it can be used to wide range of data. A numerical illustration has been presented to explain the working of the procedure developed in the paper.

Keywords : Sensory evaluation; Chi-square; Paired comparisons.

Introduction

One is usually concerned with a set of stochastic vectors in the statistical analysis of sensory data on multi-characters. For example, one may be interested in selecting a product on the basis of its over all performance based on texture, colours, tastes, flavours etc. In univariate theory of non parametric methods, considerable amount of work has been done on the development of statistical inference procedures which remain valid for broad families of underlying distributions. In case of multi-character sensory evaluation, the need of such procedure is felt more strongly because the possibility of departure from normality becomes many fold due to large number of characters and their relationships. Moreover, even if the different variates of a stochastic vector are marginally normally distributed, their joint distribution can still be quite different from a multi-normal one. The non-parametric tests and estimates are either based on Hoeffding's U-statistics [5] (and their extensions) or on suitable families of rank order statistics. The distribution theory of U-Statistics has been studied very thoroughly and elegantly and it has also been extended in various directions by many workers. Puri and Sen [7] had presented a detailed account of the distribution theory of U-statistics and some other allied statistics. They have also studied the distribution theory of various families of rank order statistics and permutation tests. Quade [8], Puri and Sen [6] and Sen and Puri [10] have considered analysis of covariance tests in non-parametric set-ups. These tests can also be used with advantage in the analysis of growth curves.

Davidson and Bradley [2] have proposed a model for multivariate paired comparisons. The model is an extension of the univariate model developed by Bradley and Terry [1] to situations in which there is interest in p attributes or characteristics. Each response to treatment pair (i, j) consists of a vector of preferences $S = (s_1, \ldots, s_p)$ whose components s_α indicate which treatment in the pair is preferred on attribute α. Large sample properties of this model have been studied by Davidson and Bradley [3]. They have also considered a regression model for the estimation of relationship in multivariate paired comparisons. The over all quality was considered as an attribute in addition to p specified attributes and $(p + 1)$ variate regression model was used. The analytical procedures are very complicated and they require heavy computations. Solution of the normal equations needs about 8 to 10 interations for estimating the treatment parameters. In this paper, we attempt to present the multivariate analogues of the univariate non-parametric procedure for analysis of multi-character sensory evaluation in paired comparisons. The procedure is quite simple and is valid for certain broad families of underlying distributions and yet is also reasonably efficient for normal distribution.

2. Multi Character Paired Comparisons

Consider t (> 2) treatments in an experiment involving paired comparisons and the responses for each treatment have been obtained for $p(> 1)$ sensory characters. The responses are in terms of ranks obtained by the treatments for different characters. The parameters of interest are the set of rank values for various treatments and characters. The null hypothesis to be tested is that the t treatments do not differ among themselves in respect of p characters. For $t = 2$, the test reduces to the sign test and for $t > 2$, the usual chi-square type tests may be used but these tests are less efficient. There will be $t(t - 1)/2$ pairs and each treatment will be compared $(t - 1)$ times. Both the treatments of a pair are presented one after the other for quality testing and the ranks will be allotted separately for each character and the treatment which is preferred will be given rank 1 and the other treatment will have the rank of 2. The entire procedure of rankings may be repeated n times.

Let $R(X_{ijkl})$ be the rank of the jth treatment of the ith pair for the kth character in the lth replication ($j = 1, 2, \ldots, t$; $i = 1, 2, \ldots, t(t-1)/2$; $k = 1, 2, \ldots, p$; $l = 1, 2, \ldots, n$).

Sum of ranks R_j for the jth treatment will be given by

$$R_j = \sum_{l=1}^{n} \sum_{k=1}^{p} \sum_{i=1}^{t(t-1)/2} R(X_{ijkl}) \tag{1}$$

$$j = 1, 2, \ldots, t.$$

Consider the null hypothesis

$$H_0 : T_i = T_j \text{ for all } i \text{ and } j;\ i \neq j = 1, 2, \ldots, t$$

against

$$H_1 : T_i \neq T_j \text{ for atleast one } i \text{ and } j.$$

The expected value of R_j under H_0 is

$$E(R_j) = 3np(t-1)/2 \tag{2}$$

and the variance of R_j is

$$V(R_j) = n^2p(t-1)/2 \tag{3}$$

For large number of treatments, the Central Limit Theorem justifies using the standard normal distribution to approximate the distribution of random variable

$$[R_j - E(R_j)]/\sqrt{V}(R_j).$$

Therefore we could use the chi-square distribution with t degrees of freedom to approximate the distribution of

$$T' = \sum_{j=1}^{t} [R_j - E(R_j)]^2/V(R_j) \tag{4}$$

if R_j's were mutually independent. But R_j's are dependent as their sum is fixed

$$\sum_{j=1}^{t} R_j = 3npt(t-1)/2 \tag{5}$$

Friedman [4] suggested the use of the statistic $T = t - 1/t\,(T')$ which was found to be asymptotically equivalent to a chi-square random variable with $(t - 1)$ degrees of freedom. Substituting the values of $E(R_j)$, $V(R_j)$ and ΣR_j in (4), we get T' as

$$T' = \frac{2}{n^2 p(t-1)} \sum_{j=1}^{t} R_j^2 - \frac{9}{2} pt(t-1)$$

and the value of T will be given by

$$T = \frac{2}{n^2 tp} \sum_{j=1}^{t} R_j^2 - \frac{9}{2} p(t-1)^2 \qquad (6)$$

and this as a χ^2-distribution with $(t - 1)$ degrees of freedom for large number of treatments. The test statistic T can be used to test the null hypothesis H_0 against H_1 by using the critical region of χ^2 with $(t - 1)$ d.f.

3. Null Distribution of Test Statistic

The exact distribution of T can be obtained under the assumption that

SUM OF RANKS FOR THREE TREATMENTS, TWO CHARACTERS AND ONE REPLICATION

T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
4	6	8	4	7	7	4	7	7	4	8	6
5	6	7	5	7	6	5	7	6	5	8	5
5	6	7	5	7	6	5	7	6	5	8	5
6	6	6	6	7	5	6	7	5	6	8	4
5	5	8	5	6	7	5	6	7	5	7	6
6	5	7	6	6	6	6	6	6	6	7	5
6	5	7	6	6	6	6	6	6	6	7	5
7	5	6	7	6	5	7	6	5	7	7	4
5	5	8	5	6	7	5	6	7	5	7	6
6	5	7	6	6	6	6	6	6	6	7	5
6	5	7	6	6	6	6	6	6	6	7	5
7	5	6	7	6	5	7	6	5	7	7	4
6	4	8	6	5	7	6	5	7	6	6	6
7	4	7	7	5	6	7	5	6	7	6	5
7	4	7	7	5	6	7	5	6	7	6	5
8	4	6	8	5	5	8	5	5	8	6	4

each ranking within a pair is equally likely for all the p characters and n replications. There are 2 possible arrangements of ranks in a pair for a character and there would be $t(t-1)/2$ pairs in the experiment for each character. Therefore, the total number of possible arrangements of ranks for all the p characters would be $2^{npt}(t-1)/2$ and each of these arrangements is equally likely under the null hypothesis. Therefore the probability distribution of T may be found for a given number of treatments and characters merely by listing all possible arrangements of ranks and by computing T for each arrangement. For example, if the number of treatments is 3, the number of character is 2 and number of replication is 1 then there would be $2^6 = 64$ equally likely arrangements of ranks which are listed on pre-page :

The value of T as given in (6) is computed for each arrangement and the probability is obtained by adding probabilities of the arrangements which produce the same value of T. For example, the value of T works out to be 2.67 when the ranks are 4, 6 and 8 for different treatments and the set of this ranking occurs at 6 places in the table. Hence $P(T = 2.67) = 6/64$. The distribution of T is given below :

DISTRIBUTION OF T

Probability Function	*Distribution Function*
$P(T = 0.00) = 10/64 = 0.15625$	$P(T \leqslant 0.00) = 0.15625$
$P(T = 0.67) = 36/64 = 0.56250$	$P(T \leqslant 0.67) = 0.71875$
$P(T = 2.00) = 12/64 = 0.18750$	$P(T \leqslant 2.00) = 0.90625$
$P(T = 2.67) = 6/64 = 0.09375$	$P(T \leqslant 2.67) = 1.00000$

This procedure may be used to obtain the null distribution of the test statistic T for any number of treatments, replications and characters. However, for large number of treatments the test statistic may be taken to follow chi-square distribution.

4. Numerical Illustration

The procedure developed in the paper will be illustrated through the data obtained by Sadasivan, Rai and Austin [9] in a taste-testing experiment on five varieties of wheat bread A, B, C, D and E for three characters namely taste, softness and grain colour. The varieties were presented to the judges in pairs for their preference with respect to grain colour.

Thereafter the bread was prepared and presented for examination and preference with respect to taste and softness qualities. The rank 1 was allotted to the bread which was preferred and 2 was given to the other member of the pair and rankings for each character was done separately. Each pair was replicated 40 times. The sum of ranks obtained by different varieties added over for all the three characters is given below :

Variety	*Sum of ranks*
A	780
B	699
C	681
D	717
E	723

For testing the null hypothesis that the performance of each variety was the same with reference to the three characters under consideration, we compute the test statistic T given at (6). The value of T computed from the above data works out to be 0.46 which is distributed as χ^2 with 4 degrees of freedom. This value is statistically not significant and we may conclude that the varieties do not differ among themselves with respect to the three characters under study.

5. Conclusions

The importance and preference of a product depend on the qualities of various sensory characters. The over all quality of a product should be taken into consideration while recommending for use. The random variables obtained for various characters are supposed to have some joint probability distribution. In the parametric case, it is assumed that these probability distributions are multivariate normal. There is sufficient reasons to believe that this assumption has some serious limitations. In practice the form of the underlying distribution is very seldom known and also in many cases there may be clear indication that the distributions are non-normal. A non-parametric method of analysis has been suggested in this paper for sensory evaluation involving multicharacters. The procedure suggested is based on order statistics and the test statistic uses ranks of various objects compared in the experiment. The null distribution of the test-statistic has been worked out and for large sample it is distributed like chi-square. The method is quite simple and general and it can be used for wide range of data. The procedure has been explained through a numerical illustration.

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